IETI Transactions on Engineering Research and Practice

http://ieti.net/TERP/

2020, Volume 4, Issue 2, 17-27, DOI 10.6723/TERP.202012_4(2).0003

THE INTEGRATED DESIGN AND FABRICATION PROCESS FOR PLANAR MORPHING TESSELLATION

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Abstract In contemporary architectural practice, the tessellation of planar surfaces, such as walls or floors is usually done by using a uniform triangular, rectangular or hexagonal tile as a template. This approach, albeit the most efficient, relies mostly on colours to induce the experience of a pattern and as such is limiting. By using differently shaped tiles, the possibilities are increased, but at the cost of time and mould fabrication. In this paper, the focus is placed on integrating the design and fabrication process in order to create a morphing tessellation from differently shaped triangular tiles, by fabricating them from a single mould with adjustable edges. The design concept was based on triangular shaped tiles where the edges morph into a curved shape created by rotating the curves around the centre-point of each side of the triangle. By generating a parametric model, the possibility for creating quick design variations is introduced by adjusting the shape of the edges through predetermined parameters. The final design consisted of ten different tile shapes, each of which would require a separate mould. However, by developing a mould with walls that can be adjusted, i.e. bent according to a predetermined template, it was possible to fabricate all of the different shapes in plaster. The final result of the project took the form of a physical model of the chosen tessellation design with 336 plaster tiles produced with the adjustable mould. This demonstrates the possibility of incorporating interesting and more dynamic, but seemingly complicated designs in interior and exterior tiling along with an efficient fabrication process.

Keywords: Tessellation; digital design; mould; plaster; cast.

1. INTRODUCTION

Tessellation, or the tiling of a plane using geometric shapes with no overlaps or gaps, has been a part of architectural decoration for thousands of years. The origins of tessellation can be traced back to 4000 B.C. to the Sumerian civilization, and its development followed throughout the ancient world in various forms of colourful mosaics and tiling [1]. In fact, tessellation has played a large role in the decoration of buildings throughout history, reaching a particularly high level of sophistication in medieval Islamic art, most notably the Alhambra palace [2]. A visit to this palace in 1936 greatly influenced the work of Dutch graphic artist M. C. Escher [3], whose artistic contribution to this geometrical field is of great importance in the modern world. Also, in 1936, the earliest known spiral tessellation was developed by mathematician H. Voderberg, eponymously known as Voderberg tiling (Figure 1a). This type of tessellation is significant for its use of monohedral tiles to create a dynamic pattern. Later developments by B. Grünbaum and G. C. Shephard [4] made significant further progress in this field.

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(a) **(b)**

Figure 1. (a) Spiral tiling discovered by mathematician Heinz Voderberg [4, p. 521]
(b) Mosaic in the Hall of the Two Sisters, Alhambra Palace, Granada [5, p. 121].

Tessellation is found in all areas of architecture, from the simplest monohedral tiling found in daily architectural practice (interior tiling mainly of kitchens and bathrooms, as well as exterior paving), to the opulent mosaics adorning important sacral and secular buildings all over the world (Alhambra palace, Figure 1b). However, while substantial funds for important public commissions allow for a greater scope of artistic freedom resulting in more elaborate and interesting designs, more often than not, budgetary constraints limit possibilities in this field. In contemporary architectural practice, the tessellation of planar surfaces, such as walls or floors in interiors and exteriors is usually done by using a uniform triangular, rectangular or hexagonal tile as a template. This approach, albeit the most efficient, relies mostly on colours to induce the experience of a pattern and as such is limiting. By using differently shaped tiles, the possibilities are increased, but at the cost of time and mould fabrication. Adjustable moulds are a production method which is gaining traction for its advantages in overcoming these limitations. For example, research conducted by van der Weijst [6] saw the development of an adjustable mould for the casting of glass voussoirs of varying geometries, applied on a tessellating shell structure.

The focus of this paper is on the decorative application of tessellation within architecture and interior design. In this research, the aim was creating a morphing tessellation from differently shaped triangular tiles, while integrating the design and fabrication process in a time and cost-efficient manner. By casting the tiles from a single mould with adjustable edges, the expense of mould fabrication could be reduced. This would ultimately allow for greater possibilities in contemporary practice, creating more dynamic and elaborate designs for commonplace spaces.

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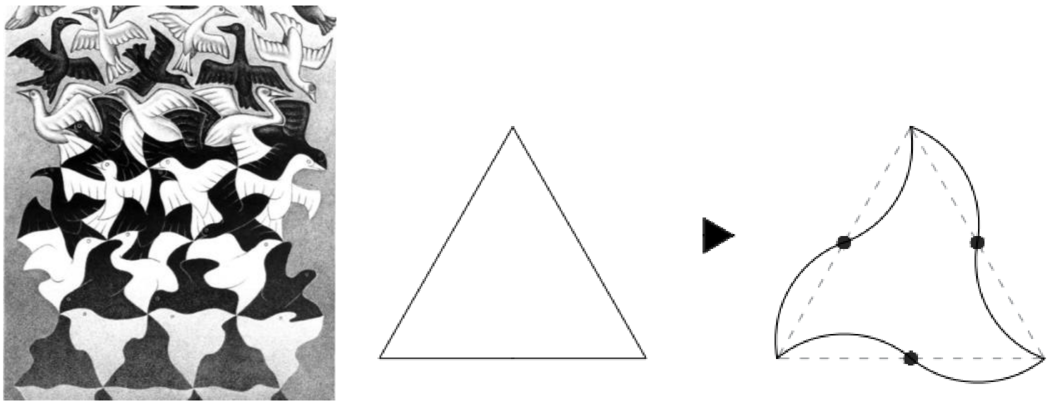
2. THE INTEGRATED PROCESS

The integrated process merges the design and the fabrication process into a complete whole. It implies adapting both of these processes to the other in the aim of achieving optimal results. The first phase of research consisted of exploring the different results that could be obtained by varying the basic parameters of tessellation – the prototile and type of transformation. The aim was to create a dynamic design through geometric transformation, but one that was practical enough for a feasible fabrication process. The second phase involved realizing the design in a cost and time efficient way. In this paper, both processes will be broken down as separate subheadings, explaining the complete design process from conceptualizing the design to its rationalization for the fabrication process, and finally the fabrication process itself.

2.1. The Design Process

The design developed for this research drew inspiration from M. C. Escher's tessellation "Liberation" (Figure 2a). Often depicting animal and human motifs in his designs, Escher gave a new lease of life to geometry-based tessellation. The design created for this research represents a reduced geometric interpretation of Escher's design, with the aim of fabrication for practical use. A triangular grid was chosen as the foundation, which gradually transitioned to a curved form reminiscent of the bird in "Liberation". The form was obtained through rotational symmetry with the midpoints of each side of the triangle being the rotational centre (Figure 2b).



(a) (b)

Figure 2. (a) Liberation by M.C. Escher, Lithograph (1955) [7, p. 28]
(b) Transformation of initial shape into curved variant in this research.

In order to allow for a larger number of variations in a time efficient manner, the possibilities of a parametric design approach were explored using Rhinoceros and Grasshopper. By using simple affine transformations and a set of actions, it enabled the design process to be observed both qualitatively and quantitatively at the same time. In generating a parametric model in Grasshopper, the possibility

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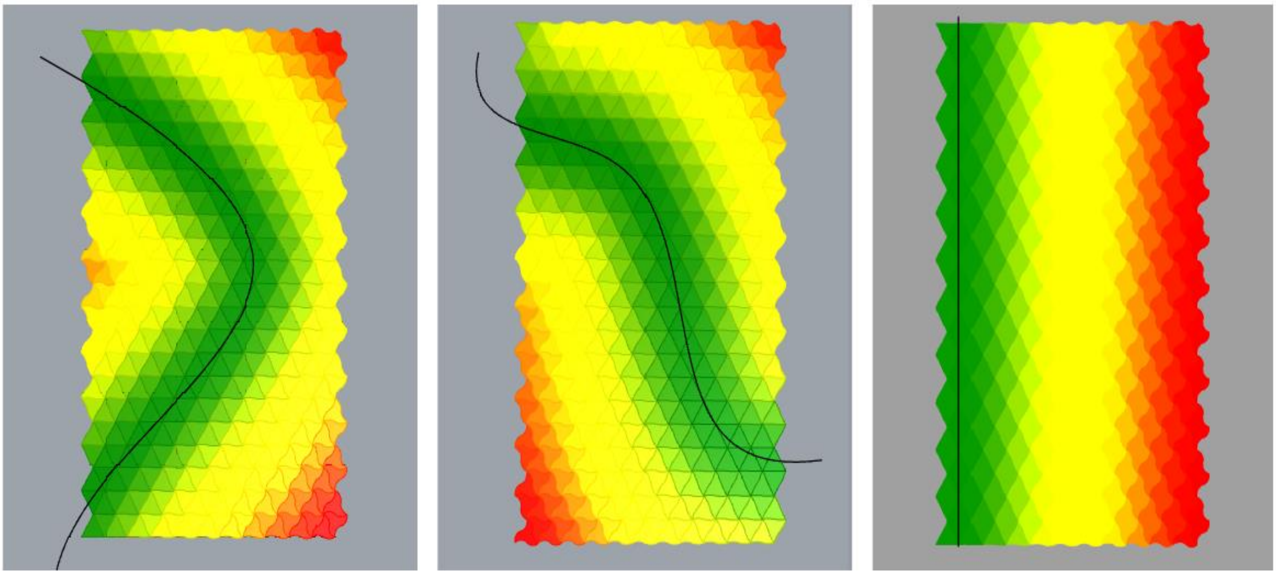
for creating quick design variations was introduced by adjusting the shape of the edges through predetermined. The most important one was the introduction of a Bézier curve. By varying its tangent, the shape of the Bézier curve was impacted which enabled different aesthetic qualities to be investigated. Additional parameters will be explained in more detail in the following section. Upon finalizing the final shape, the next step consisted of determining the rule according to which the triangle morphed into its curved variant i.e. the final shape. A curve attractor was chosen as the determinant of this rule.



Colour gradient enabling clearer visibility of the morphing pattern. Green tiles represent the initial triangular shape, while red depicts the final curved variant. The shades in between represent the transitional shapes.

Figure 3. Manipulation of the design with various forms of curve attractor, resulting in different patterns.

The edges closest to the curve attractor retained the initial triangle shape, while the edges furthest away had the final shape, with a set of transitional shapes in between. Various forms of a curve attractor (depicted as a black curve) were explored, before settling on the final design which demonstrated a proportionate ratio of the different shapes and dynamic aesthetic qualities (figure 3). The result was a tessellation which gradually morphs from triangular shaped tiles into the curved variant with parametrically controlled edge curves.

Grasshopper proved to be a valuable tool in aiding the design process, generating results of adjustments in real-time. The different parameters Grasshopper enabled to be adjusted included changing the size and number of tiles in x and y direction, the shape and curve degree of the Bézier curve and the shape and position of the curve attractor. These variable parameters allowed for the possibility of the final design being adapted for application on different dimensions of surfaces, as well as creating different visual effects. Once the triangular pattern was chosen it was necessary to make certain adjustments before proceeding to the fabrication phase.

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2.2. Adapting the Design for Fabrication

At this point, the design consisted of a large number of different shapes. The aim of this phase of research was to adapt the design for a more efficient fabrication process without compromising the aesthetic qualities. This was done by reducing the amount of differently shaped tiles to a viable number for practical application, but which was ultimately high enough to retain the dynamic character of the morphing design.

By rationalizing the curve extremes - the maximum distance curve points can have in reference to the initial straight edge of the tiles i.e. restricting it to certain intervals, the number of different tiles was reduced to ten. The intervals were marked by the numbers 0 through 3, 0 being the straight edge of the triangular tile, with a value of 0mm, and 3 the edge with the greatest curve extreme at 11mm, while 1 and 2 were linearly interpolated. By using basic Grasshopper tools, it was possible to examine all tiles generated in such a manner, create an annotation based on the edge shape and label each triangle accordingly (Figure 4) so the proper number can be noted and later on fabricated with the draft of implementation to follow. In addition, a list of the number of each combination was generated, further aiding the logistical aspect of preparing the design for fabrication.

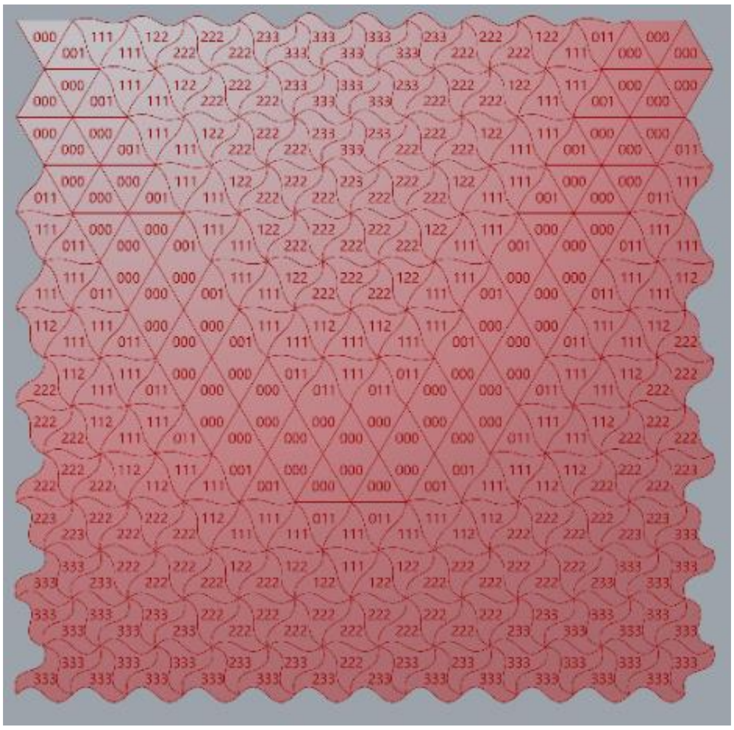

Figure 4. Final design in Rhinoceros with tiles labelled accordingly.

3. PRACTICAL APPLICATION

The final design consisted of ten different tiles, each of which would normally require a separate mould. However, the development of a mould with walls that can be adjusted i.e. bent according to a

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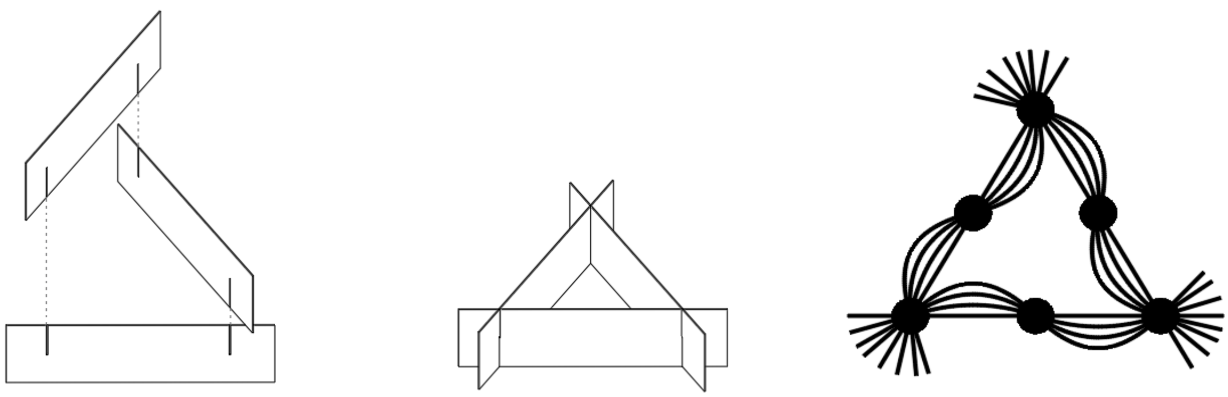
predetermined template opened the possibility of producing all of the different shapes in one mould. The aim of this phase of the research was to devise a mould which could successfully be used to produce all ten of the tiles. The mould developed consisted of two components – a fixed plywood base and adjustable walls made of ABS.

3.1. Mould Development – Base

The mould was conceived around the idea of inserting the adjustable ABS walls into grooves engraved in the plywood base, forming a perimeter for the casting of the tiles. As the mould needed to allow for the fabrication of all the different tiles, the first step in creating the mould was to overlap the shapes labelled 000, 111, 222, 333, as these can be considered the baseline shapes from which the other variants can be obtained. The chosen length of the sides of the equilateral triangle prototile was 8cm. On overlapping, the shapes converged in all three corners of the initial triangle shape. Furthermore, the design of the mould was devised to accommodate the interlocking of the ABS strips with a waffle joint (Figure 5a, 5b). For this, continuing each edge in the direction of the tangent of the end vertex point was necessary. To prevent the damaging of the mould i.e. chipping of the sharp corners in the converging points of the overlapping edges during use, circles were incorporated into the engraving template (Figure 5c). This proved to be effective in reducing damage to the mould in the later phase of its practical application.



(a) **(b)** **(c)**

Figure 5. (a,b) Graphic depiction of the interlocking of ABS strip walls with a waffle joint and (c) Graphic depiction of the template for engraving.

There were also two additional parameters to consider when designing the mould for fabrication which could affect its functioning. Firstly, the thickness of the grooves engraved into the plywood by a laser cutter needed to precisely correspond to the thickness of the ABS strips which were to be inserted. The strips used had a thickness of 0.45mm, thus the grooves in the plywood needed to be slightly wider, allowing some room for manoeuvring. Taking into consideration an approximate kerf width of 0.2mm on each side, the resulting width of the grooves was 0.9mm. This width proved to enable smooth insertion of the ABS strips, without being too loose. Secondly, the depth of the grooves

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was an important aspect to factor into the design of the mould. The course of the research led to the conclusion that the deeper the groove, the more securely the ABS strip remained in place. The minimal depth empirically proved from the findings is 6mm, which is the depth used in the research. Figure 6 depicts a side by side comparison of the engraved base, and the mould in use during casting upon insertion of the ABS strips.

During the fabrication process of the mould, there were certain issues relating directly to the method of engraving. In this case, the 3D engraving possibilities of the laser cutter available were used, testing its performance for a task more commonly done with a CNC milling machine. Namely, to achieve the desired groove depth of 6mm, several rounds of engraving were required. The focal length of the laser proved to be an issue, as it affected the width of the groove, making it narrower with each subsequent engraving job. This resulted in a groove with a cross-section shaped like an isosceles trapezoid - widest on the surface and tapering downwards. Inevitably, this partially affected the ability of the ABS strip to be inserted all the way through. Another issue encountered related to the build-up of residue in the grooves from the engraving process, which, due to the small width of the grooves, wasn't viable for removal.

Figure 6. Engraved base of the mould on the left, and mould with inserted walls in use on the right.

Furthermore, the mould required an additional element in order to be functional. Once the adjustable walls were set in the corresponding grooves of the base, parts of the "unused" grooves in the interior of the mould needed to be covered to prevent the plaster from seeping into these crevices. Several methods were tested to resolve this issue, including a plastic film overlay and a removable disc in the shape of the tile being made. In the end, a laser-cut acrylic disc was used for this element and proved to be favourable not only for this function, but also instead of a mould-release agent, which will be addressed in more detail in section 4.

Despite these issues during the development process, the plywood mould performed adequately in the tile fabrication phase. Further research could include exploring a different engraving method, i.e. a CNC milling machine, which would solve problems regarding the focal length or residue build-up.

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3.2. Mould Development – Adjustable Walls

The adjustable walls were the key feature in fabricating all of the differently shaped tiles from one mould. Different materials were investigated for the walls, including PVC foam boards and A-PET sheets. The material chosen was ABS edging tape for its range of thicknesses, flexible quality and shape, i.e. it is pre-cut into strips.

Initially, a thickness of 0.6mm tape was tried for the walls. However, this thickness proved to be too rigid for the relatively small dimensions of the tiles. 0.45mm thick tape resolved the flexibility issues and was easily able to be bent and inserted into the grooves of the base. Four different lengths of the tape were cut, corresponding to the lengths of each edge from the shortest to the longest and labelled L0, L1, L2, L3, L4 respectively. In order to "seal" the corners of the tile where the three walls met, slits were created through the tape. Interlocking the individual strips in a waffle joint. This type of joint proved to be an efficient method of fixing the strips in place, as well as acting as a leak-proof barrier for the tile casting process.

While the results achieved using the ABS edging tape as a wall for the mould were satisfactory, minor difficulties were encountered in the way the tape interacted with the base of the mould, mostly due to the aforementioned engraving issues. As explained in the previous section, the tapered cross-section of the grooves limited the ability of the ABS tape to be inserted completely. This caused the tape to occasionally pop out during the assembling of the mould. This affected the time required to put together the mould, but didn't create problems during the casting process. However, the limited insertion capacity of the grooves did negatively affect the tile casting process in another way. Given the 3.3cm height of the ABS walls and the fact that they weren't completely secured in the grooves, they tended to tilt outwards slightly towards the top. While this problem was not initially observed, it led to issues upon the assembling of the tiles which will be presented in the Results section.

3.3. The Casting Process

Once a mould had been developed, the next phase consisted of testing its functionality through the tile casting process. The final result of the research is presented in the form of a physical model consisting of 336 tiles of the selected tessellation design. For the material of the tiles, plaster was chosen for its favourable casting qualities, quick hardening time and light mass.

To begin with, several ratios of plaster to water were tested to determine the desired consistency. According to the manufacturer (plaster used was model plaster produced by Rigips) a plaster to water ratio of 2:1 is recommended. However, due to the thicker consistency and rapid hardening of the plaster, the ratio used throughout the casting process was closer to 1,5:1. Measurements were made approximately throughout the process, and variations in consistency weren't found to affect the aesthetic or durable qualities of the final tiles.

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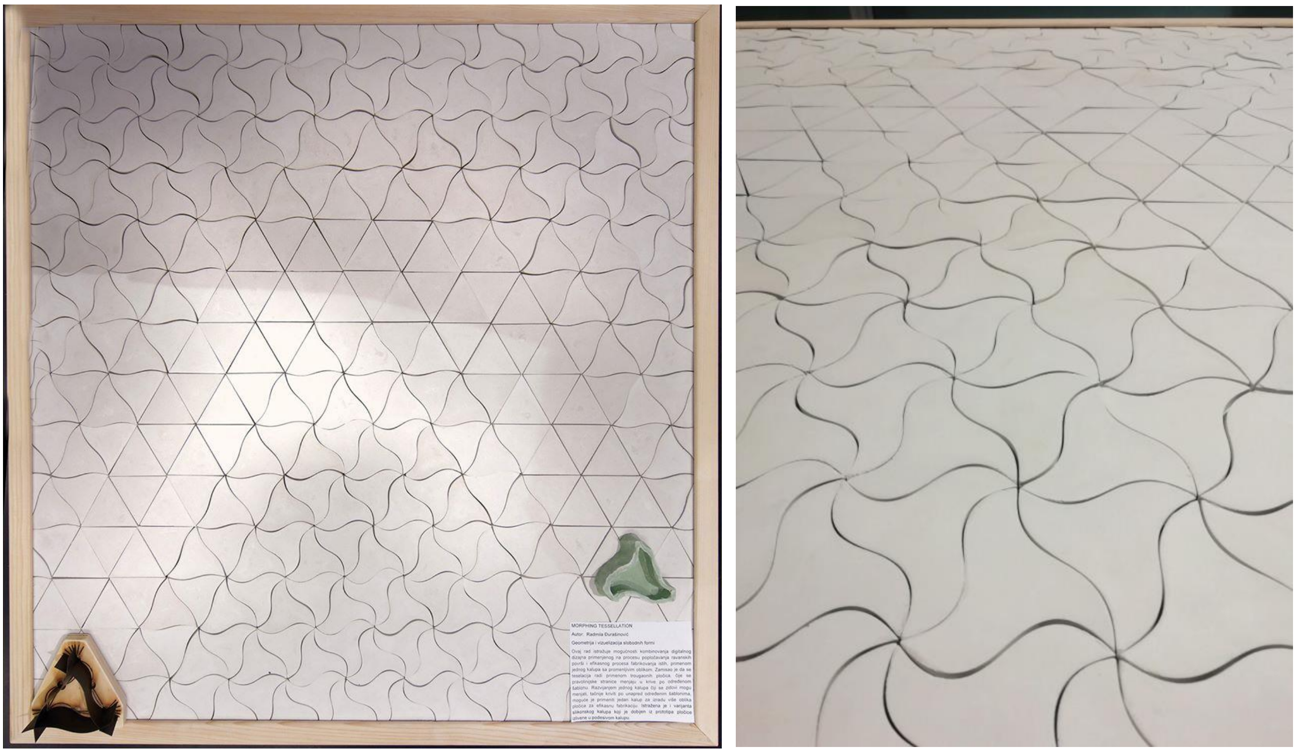
A step that can be an issue with plaster in general is releasing the form from the mould. Often, a mould release agent is used to aid this part of the casting process. However, as both the walls of the mould and the removable disc forming the base are made of plastic, this was not an issue. The smooth surface of these elements provided favourable circumstances for releasing the tiles from the mould without damaging them. Approximately 25 minutes was necessary for the plaster to harden sufficiently before removal from the mould.

Overall, the number of defective tiles was less than 10% and most likely due to several factors in play, not one in particular. Human error can be considered the cause of the majority of defects i.e. inaccurate ratio of plaster to water, less than optimal hardening time or improper handling of the mould upon tile removal. Therefore, the method can be considered adequate in producing differently shaped tiles from one mould.

In order to speed up the casting process, which in this case relied on manual labour, silicone moulds were developed from the plaster tile prototypes obtained from the adjustable mould. This proved to be effective in that it eliminated the time required to set up the adjustable walls from the process after the prototypes had been created.

3.4. Results



(a) (b)

Figure 7. (a) Final model (Image from Digital Lab 2.0 Exhibition 2019, by Vesna Stojakovic) and (b) Detailed view of interlocking tiles.

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Once all 336 of the tiles were fabricated in the adjustable and silicone moulds, the following and final step consisted of assembling the tiles according to the chosen design. The final assembled model is shown in Figure 7a. The practical application of the presented research was a success in that the tiles were easy to assemble and interlocked without major deviations. However, the walls of the tiles not being completely vertical with a slight tilt did cause slight variations in the width of the gaps between tiles (Figure 7b).

This stemmed from the aforementioned issues regarding the tilt of the adjustable walls of the mould. Overall, the entire composition was able to be assembled with minor discrepancies in fit, resulting from the handmade nature of the practical research.

4. CONCLUSION

The results of the research showed that Grasshopper is an invaluable tool for aiding the design process. It allowed for easy and quick changes to the initial design, as well as rationalizing the design for preparation for fabrication. In addition, the adjustable mould developed was successful in producing the differently shaped tiles. The research process showed that the thinner ABS edging tape proved more effective, without compromise on sturdiness. Furthermore, plastic in general, was a good choice of material in combination with the plaster as it provided a smooth base and easy release for the tiles. The project resulted in an aesthetically pleasing design using geometry as the sole variable parameter and was successfully fabricated in the form of a model, integrating these two processes for more efficient use in the fields of architecture and design in future.

The varying parameters set up in Grasshopper make the design easily adjusted for future research or projects, making it adaptable for a wide range of surface dimensions and visual effects. However, further investigation of different engraving methods could lead to improvement. Future research could involve testing the design of the adjustable mould base using a CNC milling machine as the engraving method, and seeing whether it solves problems regarding the focal length or residue build-up.

Acknowledgements

This research (paper) has been supported by the Ministry of Education, Science and Technological Development through the project no. 451-03-68/2020-14/200156: “Innovative scientific and artistic research from the FTS (activity) domain” and the University of Novi Sad, Faculty of Technical Sciences - Department of Architecture and Urban Planning.

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